MAE 4272: Fluids and Heat Transfer Laboratory

Blade Design Project - Final Report

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Group 411- #2

Executive Summary

Context and Objectives

The team designed a blade based on NACA 4412 airfoil geometry, with a final length of 6 inches that had optimized chord length and pitch angle. Maximum power generation was the team's ultimate goal, and design work was based on a far-field wind velocity of 4.73 m/s, as described by the Weibull distribution, and a rotation rate of 1200 rpm.

Design Process

The chord length and pitch angle optimization of the airfoil geometry was based on calculating the average Reynolds number across the entire blade, updating the angle of attack and ensuring it maximized the C_L/C_D ratio, and taking these parameters to derive equations for chord length and pitch angle as a function of blade radius. A CAD model was then generated with parameterization of these equations, which allowed for the model to be easily changed throughout the design process. Bending stress and torque analysis of the preliminary design was conducted and ensured that our design would not catastrophically fail during testing.

Testing Procedure

The team approached data collection through conducting an RPM sweep from 1800 - 0 RPM at selected wind speed. This data collection was then repeated for multiple wind speeds from 3.0-5.4 m/s, and took place over 2 testing days.

Discussion

The blade in experimentation generated a maximum of 3.98 W at a wind speed of 4.8 and rotational rate of 1570 rpm, and notably with a maximum torque brake of 3.6 oz-in. This was far different than the original design objectives of 4.7 m/s, 1570 rpm, and 0.0588 oz-in. Likely causes for this discrepancy include setbacks during data collection, limitations in the maximum RPM allowed to be used with the experimental setup, and issues with the torque brake. For future iterations, the blade would be designed around an optimal rotational rate of 1550 rpm and the same wind speed of 4.7 m/s, ensuring that more power will be generated for this system and loading conditions.

Conclusion

The team successfully designed the blade around the goal of maximizing power output through optimizing chord length and pitch angle. Experimentally, the blade generated a maximum power output of 3.98 W at 4.8 m/s and 1570 RPM, and the blades did not catastrophically fail during data collection. The team ultimately worked well together, with each member taking on responsibilities that aligned with their strengths. Future work includes improving data collection by including a more robust RPM sweep and updating blade parameters to better reflect the experimental rotation rate of 1570 RPM.

Context and Objectives

The group was tasked with designing a wind turbine blade with the following specifications: a maximum length of 6 inches, compatibility with a standard 1-inch radius hub, operation at a fixed angular velocity (capped at 2000 rpm), and functionality in an environment where wind speeds follow a Weibull probability distribution. After extensive discussion, the group identified the primary objective as maximizing power output through aerodynamic efficiency. With this goal in mind, we began gathering data on the operating conditions for our blade design. Using the blade's parameters, we generated the Weibull distribution and determined that the optimal wind speed for maximum power generation was 4.73 m/s, within a wind speed range of 1-10 m/s (see Figure 1). This provided a target wind speed for optimizing power output. Next, we selected a constant rotation rate and an appropriate airfoil. Drawing from Lab #6, where we tested the NACA 4412 airfoil, we found that the optimal power generation, within the turbine's 2000 rpm limit, occurred around 1200 rpm.

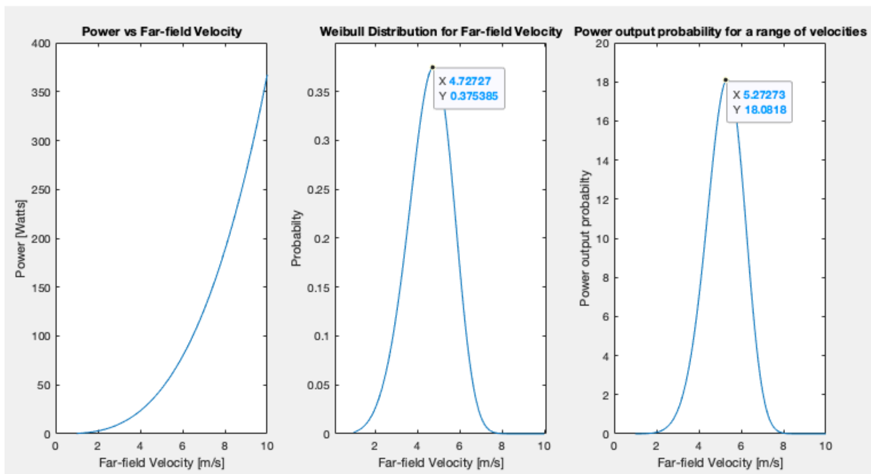


Figure 1: Weibull probability distribution for wind speed range 1-10 m/s

Design Process

Design Optimization for Pitch and Chord

When designing the blades, we undertook the task of optimizing the chord length and pitch angle for the blades. To best understand our finding, we must review the relevant equations utilized.

Fundamental Building Blocks

For the following variables, we analyzed them as functions of r or radial distance from the blade hub.

The magnitude of the effective velocity ($U_{eff}(r)$). The effective velocity vector consists of an axial component governed by the far field velocity and axial induction factor. Perpendicular to this is a tangential component governed by the angular velocity, radial distance r, and the angular induction factor. We find this portion particularly fascinating as it is encountered with respect to a blade fixed frame due to the blade's rotation rather than any actual wind speed. In addition, the angular induction factor (a') is designed to account for the manner in which the incoming air deflects immediately before arriving at the leading edge influencing a twisting effect when considering the entire turbine.

$$U_{eff}(r) = \sqrt{\underbrace{\big(\underset{\text{Far Field Velocity}}{U_1}(1-\overset{\text{axial induction}}{a})\big)^2}_{\text{axial component}} + \underbrace{\big(r\underset{\text{Angular velocity}}{\Omega}(1+\overset{\text{angular induction}}{a'})\big)^2}_{\text{tangential component}}}$$

Figure 2: U_eff equation

The pitch angle (Theta(r)) is difference between an angle (Phi) and the angle of attack (Alpha) where Phi is the angle pitch angle needed to align the blade exactly effective velocity vector and is found by taking the inverse tangent of the axial velocity over the tangential velocity (velocity in the plane of rotation). The angular induction factor is present in the denominator of the inverse tangent requiring a smaller pitch angle corresponding a greater blade alignment in the tangential direction to match the effective velocity vector than if its effect was discounted.

$$\theta(r) = \phi(r) - \alpha$$

$$\theta(r) = \tan^{-1}\left(\frac{(1-a)U_1}{r\Omega(1+a')}\right) - \alpha(r)$$

Figure 3: Pitch angle equation

The chord length (C(r)) was found using blade element method. Analyzing the axial force on some blade element of differential thickness a distance r (denoted dT(r)) we can equate the axial force via energy/momentum conservation and airfoil theory to evaluate the chord length.

$$\rho U_1^2 4a(1-a)\pi r\,dr = dT(r) = 3\Big[\underbrace{\tfrac{1}{2}C_L\rho U_{eff}^2 c(r)\,dr\cos(\phi)}_{dL} + \underbrace{\tfrac{1}{2}C_D\rho U_{eff}^2 c(r)\,dr\sin(\phi)}_{dD}\Big]$$

$$C(r) = \frac{4U_1^2 a(1-a)\pi r}{3\left[\frac{1}{2}C_L U_{eff}^2\cos(\phi) + \frac{1}{2}C_D U_{eff}^2\sin(\phi)\right]}$$

Figure 4: Chord length equation

Reynolds number (Re(r)) incorporates the magnitude of the effective velocity and chord length to provide a non-dimensional parameter for every cross section along the blade. This value (or its average) is used to iteratively optimize the chord length equation and finally provide the basis for an angle of attack function as a function of radial distance.

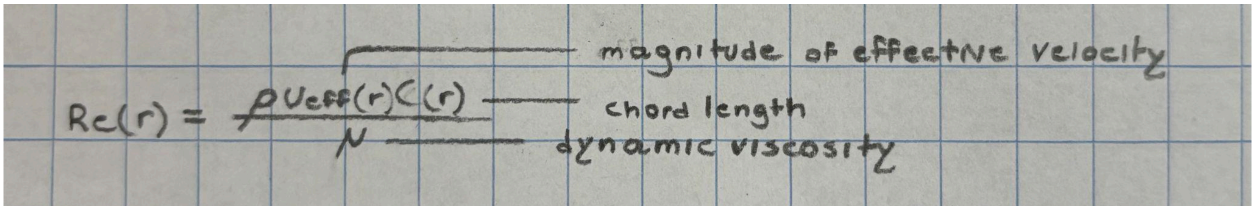


Figure 5: Reynolds number equation

The Optimization Process

- The Circular Problem

Previously, in Lab 5 we assumed a constant Reynolds number of 100,000. This influenced our decisions to use a constant value for Angle of Attack for our pitch angle calculations and lift and drag coefficients for our chord length calculations that correspond to maximum the CL/CD ratio for the Reynolds number.

But… Reynolds number is not constant for a given cross section some radial distance r from the hub. It depends on chord length, but chord length depends on it. If we are to consider changing Reynolds number, the ideal angle of attack varies as does CL and CD, all of which become functions of r or radial distance from the blade hub.

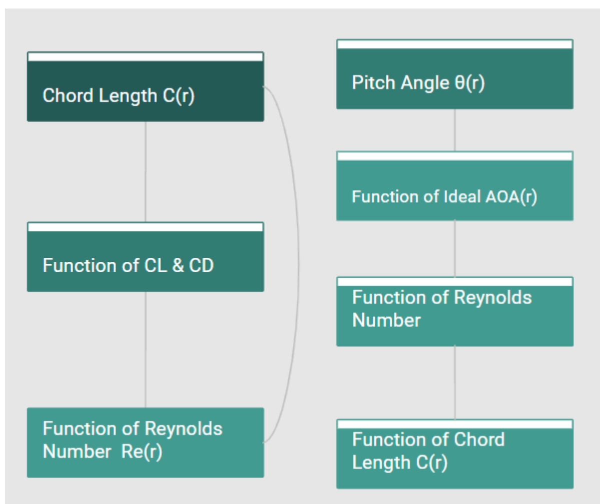
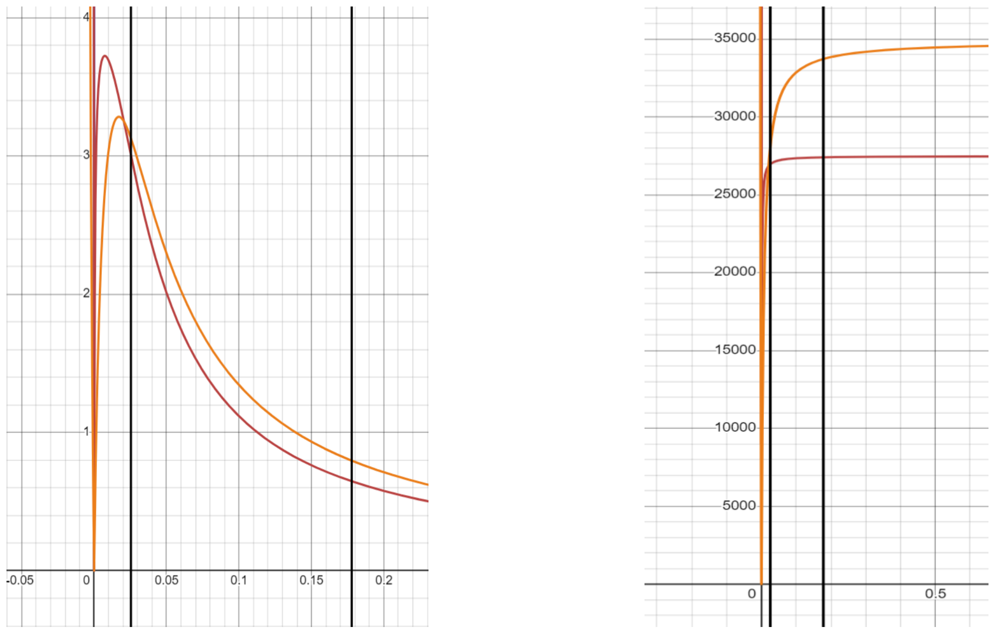


Figure 6: Depiction of circular problem

- Optimizing Chord Length

In our first iteration, CL and CD correspond to the angle of attack 6.75 degrees where the CL/CD ratio was maximized for a NACA 4412 airfoil with a Reynolds number of 100,000. We optimize the Reynolds number iteratively by calculating its average across the blade noting that its calculation or function encompasses the chord length and effective velocity functions. Interestingly, the Reynolds number function denoted $Re(r)_1$ was relatively constant with Re_{MIN} = 26949 occurring at r = 1 inch, Re_{MAX}

occurring at r = 7 inches, and the average value being 27300. For this newfound average Reynolds number, we note the angle of attack 10.5 degrees maximizes CL/CD and replace CL and CD accordingly in the chord length equation obtaining $C(r)_2$. We iterated this process twice. Below are the graphs of both iterations of the Reynolds number and chord length as functions of radial distance.



2nd Chord Length (Orange) versus First Chord Length (Red) in inches as a function of radial distance in meters Figure 7.a	2nd Reynolds Number (Orange) versus 1st Reynolds Number (Red) as functions of radial distance in meters Figure 7.b

Figure 7: Chord and Reynolds number plots

- Chord Length to Pitch Angle

With our updated chord length equation $C(r)_2$ factored into our Reynolds number equation $Re(r)_2$, we can then find the approximate Reynolds numbers and corresponding angles of attack (for which CL/CD is maximized) for radial intervals or blade sections. See figure (2) and figure (3) in the supporting documentation. We then make a plot of the optimal angle of attack and radial distance (r) and perform a linear regression obtaining alpha(r) or the angle of attack as a function of radial distance. This was found to be (alpha(r) = 0.017r^2 - 0.247r + 10.456) using the graph as shown below in figure (7). We finally add alpha(r) to the pitch angle equation theta(r).

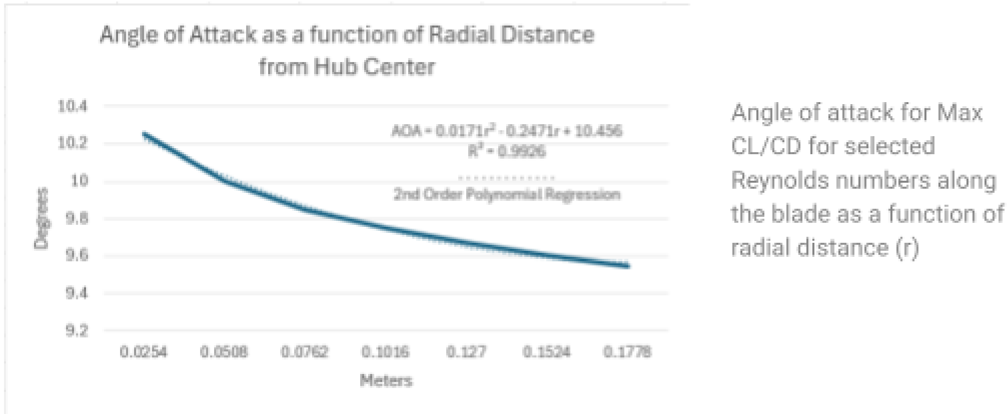


Figure 8: Angle of attack for Max CL/CD

The figure (6) shown below is a graph of phi and the updated pitch angle and figure (4) in the supporting documentation provides a list of equations graphed on Desmos and used for the entire optimization process.

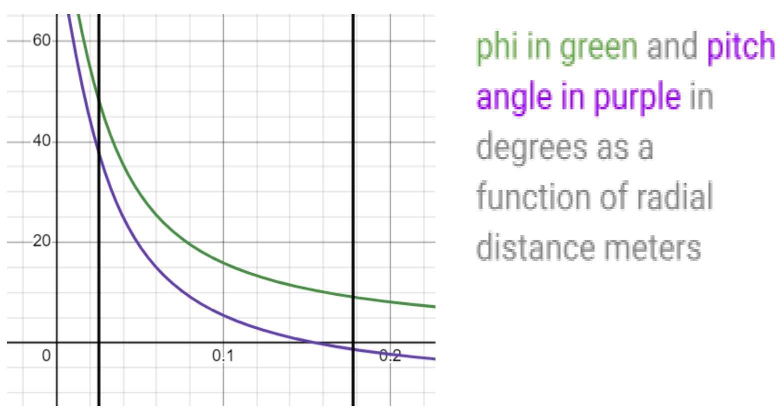


Figure 9: Phi and Pitch angle vs r

Wind Turbine Design

After deriving equations for chord length and pitch angle as a function of blade radius, discrete values were calculated for various radii using excel sheets. With planar geometry dictated by airfoil shape, chord length determining scale, and pitch angle determining orientation, we were able to completely constrain our turbine design across various radii.

As our group had limited time to design our turbine blade, we decided to design our blade using parametric variables for cross section radii, pitch angle, and chord length at each turbine cross section of interest. This enabled our group to rapidly iterate through many designs as we continued to optimize our blade design. This allowed us to not only fix mistakes as they occurred, but to also begin working on our CAD design before we finished our mathematical model and analysis. This allowed our team members to work in parallel, rather than having workflow blockages in which many group members were waiting for a teammate to finish their part to begin starting the next. Through this parallel workflow, not only were we able to work more efficiently as a group and fit more time into our busy, and often conflicting schedules, but we were also able to dedicate more time to each specific aspect of our design since we were not constrained by the chronological design constraints as our peers.

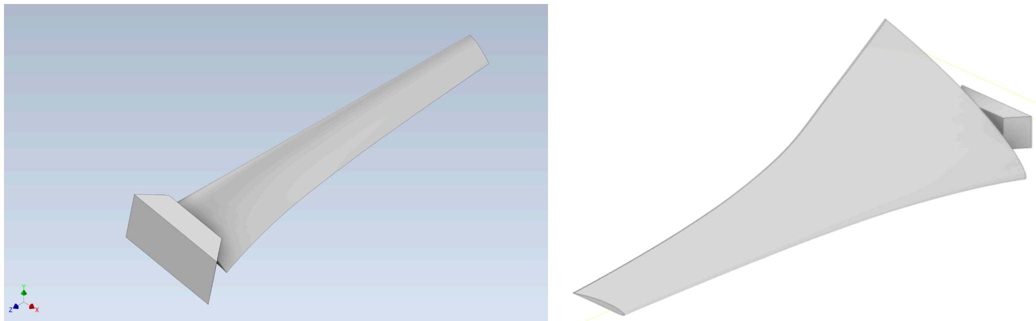


Figure 10: Preliminary Blade Design CAD vs Final Blade Design CAD

This parametric design methodology also served other practical purposes. First, we were able to iterate designs with relative ease. We were able to make rapidly different designs by altering a few global variables in the Inventor Parameter Menu. The sweeping changes in our designs can be seen by comparing our Initial Design (See Figure 10) as submitted with our Progress Report with the Final Turbine Design. Notably, the variance in chord length and pitch angle is significantly higher in our final design over our initial design with $\Delta C_{initial} = c_i(1in) - c_i(7in) = 1.03in$ and $\Delta C_{final} = c_f(1in) - c_f(7in) = 1.99in$. Across both designs, the chord length difference nearly doubled. Despite this drastic design change, the new design implementation only took ~15 minutes, thanks to the added effort in parameterization earlier on.

Another boon from design parameterization came during the design sanity checklist prior to our final submission. Upon reviewing our turbine, we realized that our pitch angle was changing opposite our desired direction. This crisis was similarly averted very quickly by changing the multiply of all the pitch angle parameters by -1, fixing a disastrous design mistake in 2 minutes instead of 2 hours.

Structural Design

To ensure that the turbine design would be successful structurally, we calculated the maximum torque and maximum bending stress associated with our chosen geometry and compared these values with the maximum bending stress of the 3D print material (PLA) and the maximum torque allowed for the B5Z model.

Following examples from literature, we approximated the turbine as a simply supported cantilever beam with a rectangular cross section. However, in an analysis done by de Lannoy[1] and Besnard et al.[2], a

[1] S. de Lannoy, "Section Modulus and Bending Inertia of Wings," [Online]. Available: http://www.wingbike.nl/Wingbike_Hydrofoil/Background_files/Section%20Modulus%20and%20Bending%20Inertia%20of%20Wings.pdf. [Accessed: Dec. 18, 2024].

[2] E. Besnard, A. Schmitz, K. Kaups, G. Tzong, H. Hefazi, H.H. Chen, O. Kural, and T. Cebeci, "Hydrofoil Design and Optimization for Fast Ships," Proceedings of the 1998 ASME International Congress and Exhibition, Anaheim, CA, November 1998.

correction factor of 0.75 and 0.85 was applied to the chord and the thickness in order to approximate the foil cross section by a rectangle. We decided to investigate the bending stress calculated including this correction factor, as well as not including the correction factor, and compare the two to the Young's modulus (maximum allowable bending stress) for PLA. Should either of the bending stresses exceed PLA's Young's Modulus of 3.5×10^9 MPa, we would go back to our design and choose an alternative cross section geometry or change the length of our blades.

Using the following known beam bending equations, we generated the bending stress for the turbine as a function of the blade chord as seen in Figure x:

Equations:

$\sigma = \frac{My}{I}$. where $I = \frac{chord(thickness)^3}{12}$

$\sigma_{besnard} = \frac{My}{I_{besnard}}$, where $I_{besnard} = \frac{0.75(blade\ chord)0.85(blade\ thickness)^3}{12}$

and M is found through an iterative process by finding the moment at every blade cross section using the axial force found in the Power Calculation Matlab Script and $M = axial\,force \times chord\,from\,n - 1\,to\,n$, (where n ranges from 0 to blade length, essentially getting the "instantaneous" chord location).

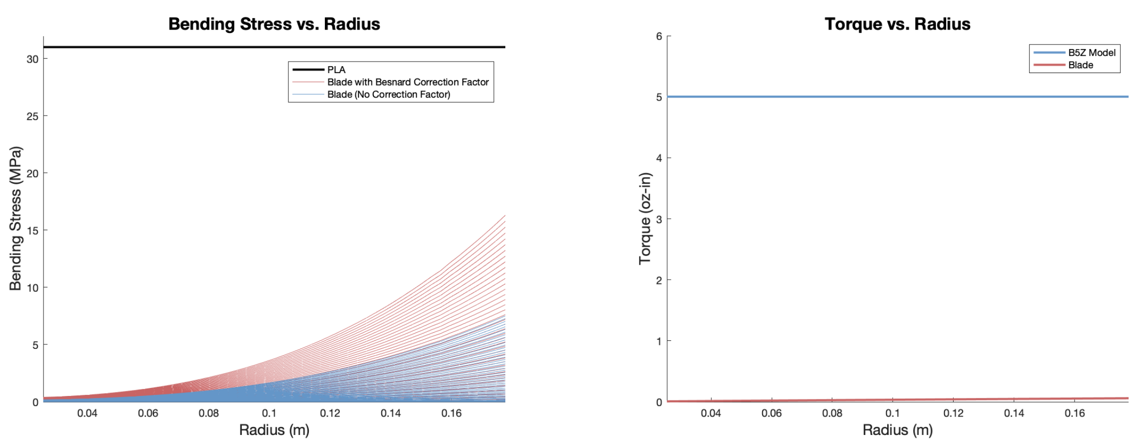


Figure 11: Bending Stress vs Blade Radius (left) and Torque vs Blade Radius (right)

The blade bending stress calculated was 7.51 MPa and considering the Besnard Correction Factor, the blade bending stress was 16.3 MPa. This is far below the PLA requirement of 31 MPa. It is clear from the bending stress analysis that regardless of whether or not we take the Besnard Correction Factor into consideration when approximating the cross section of the blade, we will be sure that our blade will not catastrophically fail under the loading conditions.

To see if the torque generated by our blade would meet the torque brake specifications, we used the axial force calculated in the Power Calculation Matlab script (as mentioned above), which is stored in dT in the original script. Plotting this and the maximum torque allowed by the torque brake (5 oz-in) yielded the torque graph as seen in Figure 11 and we found that the blade's maximum torque was 0.0588 oz-in. Again, it is clear from this torque analysis that our blade's torque is far below the maximum torque allowed by the B5Z model, meaning it can be safely stopped by the torque brake and will not catastrophically fail under the loading conditions.

After confirming that a blade could safely operate under the loading conditions, we were certain that we did not need to make any adjustments to the geometry, and we proceeded with testing our blades as described below. Please see the supporting documentation for the final Matlab script used to analyze the bending stress and torque as seen in Figure 11.

Testing Procedure

On data collection day, we had the first time slot in the class, which led to some challenges with setting up the turbine. The teaching assistants worked on the setup for a while, but it wasn't until the Professor intervened that the turbine began to operate correctly. Unfortunately, the setup process took longer than anticipated, and we were only able to collect one set of data points during this initial session. Thus, we returned for a second time slot to complete the remaining data collection.

For our approach, we decided to conduct an RPM sweep. This involved selecting a constant wind speed and starting data collection around the turbine's maximum angular speed (~1800 RPM). We then incrementally increased the torque brake, recording data points as the brake applied more and more torque to counteract the rotational motion generated by the wind turbine. We continued this process until the turbine approached a stall point and the blades came to a complete stop.

During this process, we observed an interesting trend: as the fan frequency/wind speed increased, the threshold at which the turbine's approached the stall point also shifted upwards. To investigate this further, we performed a final test at a lower wind speed to observe the stall point. As a result, we collected data at the following wind speeds: 3.0, 4.0, 4.7, 4.8, and 5.4 m/s.

Discussion

Based on our mathematical model and design optimization, our wind turbine was intended to perform optimally at 1200 RPM and a wind speed of 4.7 m/s. During our experiment, we observed a max power generation of 3.98 W at a wind speed of 4.8 m/s and a rotational rate of 1570 RPM. We observed similar torque behavior, with maximum values in the 1300-1600 RPM range across wind speeds. We observed a max torque value of 3.6 oz-in at a wind speed of 4.8 m/s. Our blade was able to perform as intended without damage to itself, the wind tunnel, or torque brake system, satisfying our validating our structural requirements and satisfying the structural constraints.

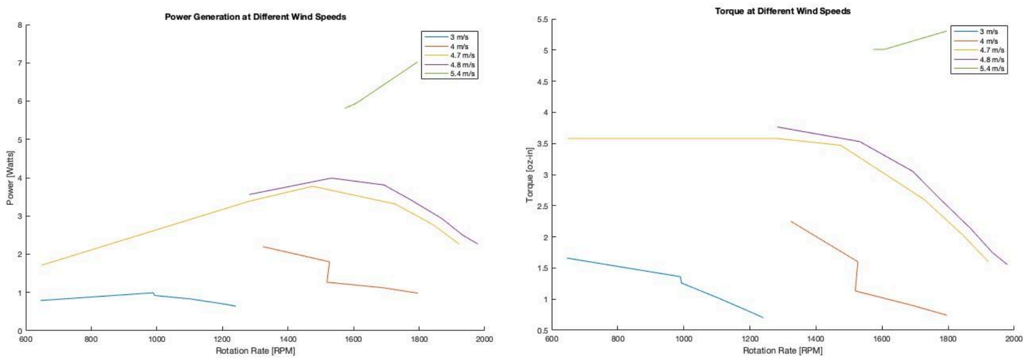


Figure 12: Power Generation and Torque Output across Various Wind Speeds

We encountered various issues and setbacks during our data collection process that we could ideally adapt to in a future design cycle. Our primary issue with data collection arose with the very tight window of rotational rates in which we were actually able to collect data. To ensure the safety of the torque-brake system, we were forbidden to collect data at rates greater than 2000 RPM. This was difficult to achieve since the turbines would naturally accelerate far past this point, even at low wind speeds, thus there is a great swath of data points in which we were unable to engage with. Another difficulty was to actually overcome the friction of the torque brake. This made it very difficult to actually get the blade spinning initially, but also greatly influenced the stall point of our blade, artificially capping the lower end of rotational rates in which our blade could function. With these factors limiting both the upper and lower range of operational rotational rates of our blade, we were unable to collect data for higher speeds in which our blade may have performed better and we were also unable to much data near or below 1200 RPM, the target rotational rate we designed our blade for.

In future iterations, we would ideally be able to test our turbine at rotational speeds greater than 2000 RPM so we could better characterize our existing blades. Armed with the knowledge we now have about the new torque brake system, we would design our blade for an optimal rotational speed of ~1550 RPM so we would be able to actually collect data around our hypothetical optimal state.

Conclusion

Our wind turbine blade design successfully met the goal of maximizing power output by optimizing key parameters such as chord length, pitch angle, and Reynolds number. Through a series of iterative calculations and design adjustments, we predicted that the optimal performance would occur at a wind speed of 4.7 m/s and a rotational rate of 1200 RPM. While testing revealed some challenges, particularly with the torque brake system and the limited range of rotational speeds we could collect data at, our blade still generated a maximum power output of 3.98 W at 4.8 m/s and 1570 RPM, confirming that our design was sound both aerodynamically and structurally.

Reflecting on our group's approach, we took advantage of each team member's strengths, allowing us to work efficiently and capitalize on our individual skills to improve the overall effectiveness of the project. The parametric design methodology, in particular, enabled us to work in parallel, making rapid design adjustments and troubleshooting in real time. This not only enhanced our workflow but also allowed us to manage time effectively, ensuring we could address challenges as they arose. However, the testing process also highlighted areas for improvement, particularly in the torque brake's impact on our data collection range. Moving forward, expanding the operational range of rotational speeds and addressing friction-related issues will be key in further optimizing our design. Overall, this project laid a strong foundation for future iterations and improvements in wind turbine blade design.